MTH1030: Assignment 1, 2016

Some hard pyramids and cubes

Due in hardcopy form with your support class leader on Wednesday, 24 August 2016, 5 p.m.

The Rules of the Game

For the most part these are pretty tough Monash Science Faculty rules that apply to all assignments in maths courses at uni.

- To submit an assignment, either hand it to your support class leader during your support class or deposit it in his or her assignment box on the ground floor of the maths building (building 28). **Note that you won't be able to find your support class leader's assignment box unless you know their name (ask them or find out on our units Moodle webpage) If you do not submit your assignment in the correct assignment box, your assignment will not be marked and you will receive 0 marks on your assignment!**
- **Maximum number of marks for this assignment is 75.** This comprises 25 marks for the first question and 50 marks for the second question. How exactly marks are split up is specified in the questions. To get full marks your report To get full marks your report
  1. must be neatly presented, but does not necessarily have to be typed;
  2. must not contain any mistakes (mathematical, logical, spelling, etc.);
  3. must be self-contained, including the statement of a problems at the beginning, and answers to whatever questions were asked at the end;
  4. must be understandable without too much effort by other students taking this course;
  5. must contain a clear description of the mathematics that justifies everything you do in **full English sentences**.

A lot of this is about the way you present your work. Just to put a figure to it, you can lose up to 30% of the total marks for poor presentation. In particular, if your handwriting is illegible, whatever you are writing about will not be counted. If you have problems in this respect you may want to consider typing everything up after all.

- **Making things easier for yourself.** To facilitate some of the more complicated and repetitive calculations in our assignments you should use *Mathematica*. As a Monash student you have access to this very powerful computational maths program. To download and register a free copy of *Mathematica* for use at home follow the instructions on our Moodle website. You can even simultaneously do all the calculations and write up your report nicely formatted using this program. Check out the sample Mathematica notebook on our Moodle page.

  Some calculations are repetitive. You only have to present full details of such a calculation once, and thereafter may just list the results of other calculations of the same type.

  Often it is a good idea to include pictures to illustrate what you are doing.

  To distinguish names of vectors from numbers use bold letters if you are typing your report (e.g., $\mathbf{u} = (3, 4, 1)$), or underlined letters if you are writing your report by hand (e.g., $\underline{u} = (3, 4, 1)$).

- **Accuracy.** Some of my coordinates are rounded to five decimal places (sounds crazy I know, but ...) Your final answers should be accurate to at least two decimal places (e.g. $12.56743\ m \approx 12.57\ m$). To achieve this sort of accuracy you should only round your final results and not intermediate results.

- **Penalties for late submission.** The penalty for assignments submitted late is 10 marks per day late or part thereof. Weekends and holidays attract the same penalty as weekdays. No assignment can be accepted for assessment more than eight days after the due date except in exceptional circumstances and in consultation with Burkard. Late assignments can be e-mailed to Burkard.

  If you know in advance that you may not be able to meet a deadline please contact Burkard as soon as possible **before** the due date.

  Support class leaders are **not authorised** to approve extensions to deadlines.

- **Submit your own work.** Please make sure to submit your own work. It is okay, and I would even encourage you to discuss a problem with other students who are taking the course to try to understand how to attack it (and to avoid going completely off the mark). However, in the end you have to do your own calculations and you have to write up your own report in your own words.

  Monash University is very picky about this last point and, to make sure that you understand how serious this is, requires that you attach a completed **Assessment Cover Sheet** to the front of your assignment. Make sure that you understand what you are signing there. You can download a copy of the coversheet from the unit's website.

Use your new tools!

This assignment is all about exploring some unusual features of cubes and right angles by solving a number of problems. There are different approaches to solving these problems, including some that don't involve the new mathematical tools that we introduced at the beginning of this course. However, to make this assignment into a meaningful exercise you are encouraged to use our new tools in your solutions:

- The dot product for calculating angles and checking quickly whether an angle is a right angle.
- The cross product for calculating vectors that are perpendicular to two given vectors (or a plane) in space and for calculating areas of parallelograms, triangles, and other plane figures in space.
- Line equations for describing lines in space.
- Plane equations for describing planes in space.

.

Okay, that was all pretty serious. Now, let's have some fun!

1 Pyramids and products (25 Marks)

The three-dimensional counterparts of triangles are four-sided pyramids. We colour the four faces of such a pyramid blue, red, green and purple. Let B, R, G and P be the areas of these faces.

In the diagram we've also highlighted a blue, a red, a green and a purple vector. Let's call these vectors $\mathbf{b}, \mathbf{r}, \mathbf{g}, \mathbf{p}$. The blue vector is perpendicular to the blue face, its length is equal to the area of the blue face and the vector is pointing outside the pyramid. The same is true for the other three vector/face combos.

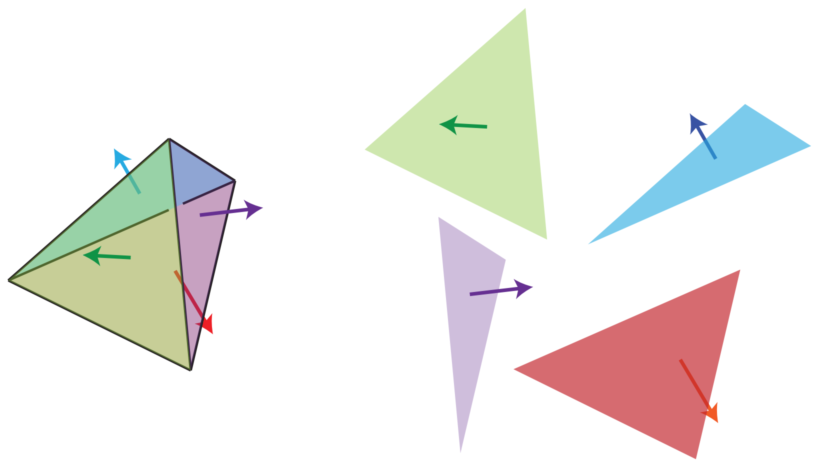

Figure 1: A four-sided pyramid and its four vector/face combos.

a) Show that

$$\mathbf{b} + \mathbf{r} + \mathbf{g} + \mathbf{p} = \mathbf{0}.$$

Hint: Start by making one of the corners of the pyramid the origin $\mathbf{0} = (0, 0, 0)$. Then the pyramid is pinned down by the vectors $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3)$ and $\mathbf{w} = (w_1, w_2, w_3)$. Now express the vectors $\mathbf{b}, \mathbf{r}, \mathbf{g}, \mathbf{p}$ in terms of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and go on autopilot.

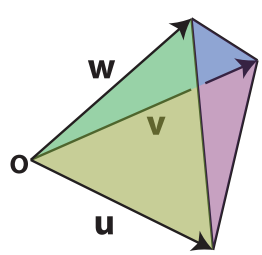

Figure 2: The pyramid is completely determined by the three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

(5 marks)

b) Using the above identity show that

$$P^2 = B^2 + R^2 + G^2 + 2\ B\ R\cos(<\mathbf{b},\mathbf{r}) + 2\ R\ G\cos(<\mathbf{r},\mathbf{g}) + 2\ G\ B\cos(<\mathbf{g},\mathbf{b}),$$

where $<\mathbf{b},\mathbf{r}$ denotes the angle between the vectors $\mathbf{b}$ and $\mathbf{r}$, and similarly for the other two angles in this formula.

Hint: Start by writing the identity as

$$-\mathbf{p} = \mathbf{b} + \mathbf{r} + \mathbf{g}$$

and then think "dot product".

(5 marks)

c) Show that

$$P^2 = B^2 + R^2 + G^2 - 2\ B\ R\cos(<BR) - 2\ R\ G\cos(<R\ G) - 2\ G\ B\cos(<G\ B),$$

where $<BR$ stands for the angle between the two faces coloured blue and red, and similarly for the other two angles in this identity. This new identity is a three-dimensional counterpart of a very famous trigonometric identity involving triangles. Which identity is that?

(5 marks)

d) Slicing a corner off a square gives a right-angled triangle, the lengths of whose sides are related by Pythagoras's theorem: $a^2 + b^2 = c^2$. Show that this two-dimensional setup generalizes to three dimensions as follows: Using a plane, slice a corner off of a cube. This sliced off chip is a special four-sided pyramid. Three of the faces are right-angled triangles, and the fourth is not. Let's call the *areas* of the three right-angled faces A, B, and C and the area of the fourth face D. Use the identities in b) or c) to show that

$$A^2 + B^2 + C^2 = D^2.$$

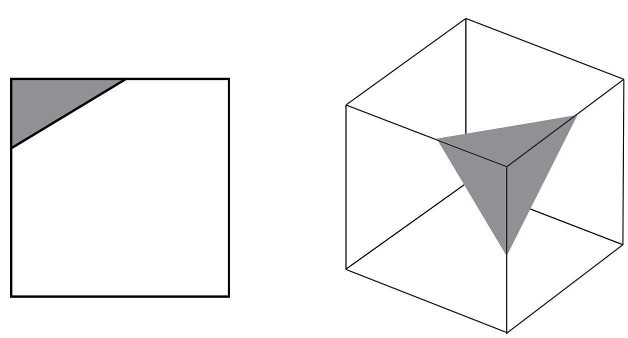

Figure 3: Slicing corners off a square and a cube.

(5 marks)

e) The funny corner sticking out of the sidewalk on Swanston Street can be considered as a corner chopped off a cube with the approximate measurements in meters indicated in the photo below. Calculate the areas of the four triangular faces of this corner.

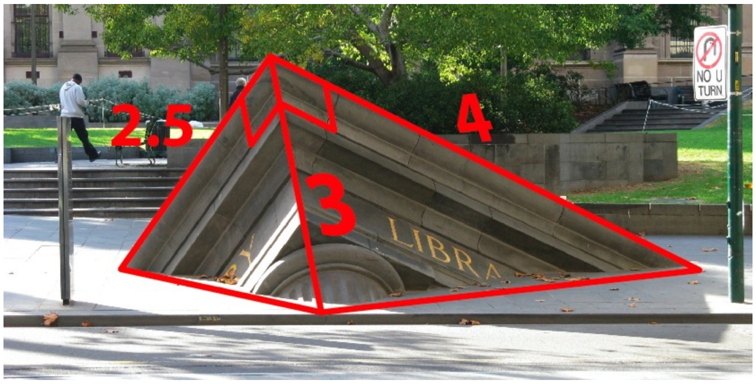

Figure 4: The funny corner on Swanston Street.

(5 marks)

2 Mapping a giant cube (50 marks)

The following pictures show a curious Rubik's-cube-like structure that greets visitors to Melbourne Museum.

Figure 5: The "Rubik's Cube" at the Melbourne Museum.

The photo on the right shows what you see when you look at the cube from above using the free program *Google Earth* (`http://earth.google.com/`). The photo on the left was taken from the ground.

Google Earth allows you to view many prominent buildings in 3d. Imagine that you are the maths wiz at Google whose job it is to create a 3d image of our cube for Google Earth. This means that you have to find coordinates of the corners of the square A, B, C, D visible from the top, as well as the coordinates of the points A', B', C', D' at which four edges of the Cube meet the ground. As indicated in the first picture, A is supposed to be connected to A' by such an edge, and similarly for B and B', C and C', and D and D'.

You are new to this job and you are supposed to base your work on your predecessor's work (who got fired yesterday). His notes are a terrible mess, but it is clear that he started modelling the cube by letting the (flat) ground coincide with the xy-plane and having the y-axis point West and the x-axis point North. His notes don't tell you where the origin of his coordinate system is supposed to be, but wherever it may be, in this coordinate system $B = (59.3506, 36.4925, 13.8151)$, $B' = (57.8976, 32.7136, 0)$, $D = (88.3907, 25.3267, 13.8151)$.

Your job also involves figuring out how long the sides of the cube are; how long the four segments AA', BB', CC', DD' are; how large the area of the quadrilateral A', B', C', D' is; and, finally, what the volume of the part of the cube that sits above the ground is.

At the end of your report please summarize your results as follows:

$$A = (*, *, *)$$
$$B = (59.3506, 36.4925, 13.8151)$$
$$C = (*, *, *)$$
$$D = (88.3907, 25.3267, 13.8151)$$
$$A' = (*, *, *)$$
$$B' = (57.8976, 32.7136, 0)$$
$$C' = (*, *, *)$$
$$D' = (*, *, *))$$
$$|AA'| = *m$$
$$|BB'| = *m$$
$$|CC'| = *m$$
$$|DD'| = *m$$
$$sidelength = *m$$
$$area(A'B'C'D') = *m^2$$
$$volume \ \ = *m^3$$

(50 marks): For any single one of these vectors/values that you get wrong 2 marks will be deducted. Tough rules: one mistake that implies other mistakes will result in every single one of these mistakes being penalized in this way—Google Earth only accepts results that work! Please make absolutely sure that you double-check your results before you submit your report!